Total No. of Questions : 8] [Total No. of Printed Pages : 4

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B.A./B.Sc. (General) Ist Semester (0001) Examination

0044

MATHEMATICS

Paper : II

(Calculus-I)

Time : 3 Hours] [Maximum Marks : 30

Note :- Attempt *five* questions in all, selecting at least *two* questions from each Section.

Section–A

1. (a) Solve for x the inequality $\frac{x+2}{n-2} < \frac{4n-1}{2n-3}$.

(b) Prove that $\left|x-\frac{1}{2}\right| < \frac{1}{3}$ iff $\frac{1}{11} < \frac{1-x}{1+x} < \frac{5}{7}$. 3,3

2. (a) State order completeness property of reals. Does the set of rational numbers possess this property ? Justify your answer.

(b) Find the least upper bound and greatest lower bound of the set $S=\left\{\frac{2-x}{1-x};\ x>0, x\neq 1\right\}$.

3. (a) Is the union of two bounded sets a bounded set ? What do you say about its converse ? Justify your answer.

(b) If $f(x)=x\left[\frac{1}{x}\right]$, does $\underset{x\to 1/2}{\text{Lt}}\ f(x)$ exist, explain your answer.

4. (a) Prove that if a function $f(x)$ is continuous at a point a and $f(a)\neq 0$, then prove that these exists some neighbourhood of a where $f(x)$ possesses the same sign as that if $f(a)$.

(b) Determine the values of a and b for which :

$$\underset{n\to 0}{\text{Lt}}\ \frac{x(1+a\cosh x)-b\sinh x}{x^3}=1$$

Section–B

5. (a) By using Lagrange's mean value theorem prove that :

$$|\sin x-\sin y|\leq|x-y|\ \text{for all}\ x,y\in R$$

(b) Calculate the approximate value of $\sqrt{24}$ to three decimal places by Taylor's expansion.

6. (a) Use Maclaurin's theorem to show that :

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots\ldots\ldots\ldots$$

$$+ \frac{(-1)^{n-1}}{n}x^n - \frac{x^n}{(1+\theta x)^n}, 0 < \theta < 1.$$

(b) Use mean value theorem to show that :

$$\frac{\pi}{6} + \frac{2n-1}{\sqrt{3}} \le \sin^{-1} x \le \frac{\pi}{6} + \frac{2n-1}{2\sqrt{1-x^2}}$$

where $\frac{1}{2} \le x < 1$. 3,3

7. (a) If $y = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\sinh^{-1}\frac{x}{a}$, show that :

$$\left(\frac{dy}{dx}\right)^2 = x^2 + a^2$$

(b) Prove that $\tanh^{-1} x = \frac{1}{2}\log\frac{1+x}{1-x}, -1 < x < 1$ and find its derivative also. 3,3

8. (a) Prove that :

$$\frac{d^n}{dx^n}\left(\frac{\log x}{x}\right) = \frac{(-1)^n \lfloor n}{x^{n+1}}$$

$$\left[\log x - 1 - \frac{1}{2} - \frac{1}{3} \ldots\ldots\ldots\ldots - \frac{1}{n}\right].$$

(b) If $y = \sin m(\sin^{-1} x)$, show that :

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 - m^2)y_n = 0$$

Hence show that :

$$y_n(0) = \begin{cases} 0, & \text{when } n \text{ is even} \\ m(1^2 - m^2)\,(3^2 - m^2)\ldots\ldots[(n-2)^2 - m^2] & \end{cases}$$

when n is odd.

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